KAP 9/12

EXPONENCIÁLNÍ ROVNICE

YESIT.CZ
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- jsou takové rovnice, kde je neznámá v exponentu

$$2^{2x-1} = 8$$

$$2^{2x-1} = 2^3$$

$$2x - 1 = 3 \quad /+1$$

$$2x = 4 \quad /:2$$

$$x = 2$$

Cíl: Zkrátit základy. Musíme mít na obou stranách stejný mocninný základ, v tomto případě dvojku.

$$\sqrt{3^x} = \sqrt[3]{9}$$

$$\left(3^x\right)^{\frac{1}{2}} = \sqrt[3]{3^2}$$

$$3^{\frac{x}{2}} = \left(3^2\right)^{\frac{1}{3}}$$

$$3^{\frac{x}{2}} = 3^{\frac{2}{3}}$$

$$\frac{x}{2} = \frac{2}{3} \quad /\cdot 6$$

$$3x = 4 \quad /:3$$

$$x = \frac{4}{3}$$

$$4 \cdot 2^{x^2} = 2^{3x}$$

$$2^2 \cdot 2^{x^2} = 2^{3x}$$

$$2^{2+x^2} = 2^{3x}$$

$$\boxed{2^a \cdot 2^b = 2^{a+b}}$$

$$2 + x^2 = 3x \quad /-3x$$

$$x^2 - 3x + 2 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2}$$

$x_1 = 2$

$x_2 = 1$

MÁ DVA KOŘENY

$$5^{x+1} - 5^{x-1} = 24$$

Pozn.: $5^{a+b} = 5^a \cdot 5^b$

$$5^x \cdot 5^1 - 5^x \cdot 5^{-1} = 24$$

$$5^x (5^1 - 5^{-1}) = 24$$

$$5^x \left(5^1 - \frac{1}{5^1}\right) = 24$$

$$5^x \left(\frac{5}{1} - \frac{1}{5}\right) = 24$$

$$5^x \left(\frac{25-1}{5}\right) = 24$$

$$5^x \left(\frac{24}{5}\right) = 24$$

$$5^x \cdot \frac{24}{5} = 24 \quad / : \frac{24}{5}$$

$$5^x = \frac{24}{\frac{24}{5}}$$

$$5^x = \frac{24}{1} \cdot \frac{5}{24}$$

$$5^x = \frac{5}{1}$$

$$5^x = 5^1$$

$$x = 1$$

COKOLIV NA NULTOU JE JEDNA, např.: $4^0 = 1$

VÝJIMKA $0^0 \Rightarrow$ ERROR

Vyřeš funkci $a = 4^b - 2$ rovnici, když $a = 0$.

$a = 4^b - 2$

$0 = 4^b - 2 \quad /+2$

$2 = 4^b$

$2 = (2^2)^b$

$2 = 2^{2b}$

$1 = 2b \quad /\cdot 2b$

$1 - 2b = 0 \quad /-1$

$-2b = -1 \quad /\cdot(-1)$

$2b = 1 \quad /:2$

$b = \frac{1}{2}$

Vyřeš funkci $a = 4^b - 2$ rovnici, když $b = 0$

$a = 4^b - 2$

$a = 4^0 - 2$

$a = 1 - 2$

$a = -1$

$$-5 \cdot 3^x + 3^x \cdot 3^1 + 54 = 0$$

$$3^x(-5+3) + 54 = 0$$

$$3^x \cdot (-2) + 54 = 0 \quad /-54$$

$$3^x \cdot (-2) = -54 \quad /:(-2)$$

$$3^x = 27$$

$$3^x = 3^3$$

$$\underline{\underline{x = 3}}$$

ŘEŠTE EXPONENCIÁLNÍ ROVNICE v R

a) $3^{2x-1} + 3^{2x-2} - 3^{2x-4} = 315$

b) $2^{x^2-6x-\frac{5}{2}} = 16\sqrt{2}$

c) $\left(\frac{4}{9}\right)^x = \left(\frac{3}{2}\right)^3$

$$3^{2x-1} + 3^{2x-2} - 3^{2x-4} = 315$$

$$3^{2x} \cdot 3^{-1} + 3^{2x} \cdot 3^{-2} - 3^{2x} \cdot 3^{-4} = 315$$

$$3^{2x}\left(3^{-1} + 3^{-2} - 3^{-4}\right) = 315$$

$$3^{2x}\left(\frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^4}\right) = 315$$

$$3^{2x}\left(\frac{1}{3} + \frac{1}{9} - \frac{1}{81}\right) = 315$$

$$3^{2x}\left(\frac{27 + 9 - 1}{81}\right) = 315$$

$$3^{2x}\left(\frac{35}{81}\right) = 315 \quad | : \frac{35}{81}$$

$$3^{2x} = 315 : \frac{35}{81}$$

$$3^{2x} = \frac{315}{\frac{35}{81}}$$

$$3^{2x} = \frac{\cancel{315}^{9}}{1} \cdot \frac{81}{\cancel{35}\,1}$$

$$3^{2x} = 9 \cdot 81$$

$$3^{2x} = 3^2 \cdot 3^4$$

$$3^{2x} = 3^{2+4}$$

$$\cancel{3}^{2x} = \cancel{3}^{6}$$

$$2x = 6 \quad | : 2$$

$$x = 3$$

$$2^{x^2-6x-\frac{5}{2}} = 16\sqrt{2}$$

$$2^{x^2-6x-\frac{5}{2}} = 2^4 \cdot 2^{\frac{1}{2}}$$

$$2^{x^2-6x-\frac{5}{2}} = 2^{\frac{4}{1}+\frac{1}{2}}$$

$$2^{x^2-6x-\frac{5}{2}} = 2^{\frac{8+1}{2}}$$

$$2^{x^2-6x-\frac{5}{2}} = 2^{\frac{9}{2}}$$

$$x^2-6x-\frac{5}{2} = \frac{9}{2} \qquad / + \frac{5}{2}$$

$$x^2-6x = \frac{9+5}{2}$$

$$x^2-6x = 7 \qquad / -7$$

$$x^2-6x-7 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x_{1,2} = \frac{-(-6) \pm \sqrt{(-6)^2-4\cdot 1\cdot(-7)}}{2\cdot 1}$$

$$x_{1,2} = \frac{6 \pm \sqrt{36-4\cdot(-7)}}{2} = \frac{6 \pm \sqrt{36-(-28)}}{2} = \frac{6 \pm \sqrt{64}}{2} =$$

$$= \frac{6 \pm 8}{2} \begin{cases} x_1 = 7 \\ x_2 = -1 \end{cases}$$

ŘEŠENÍM JSOU DVA KOŘENY TÉTO EXPONENCIÁLNÍ ROVNICE

$$\left(\frac{4}{9}\right)^x = \left(\frac{3}{2}\right)^3$$

$$\left(\left(\frac{2}{3}\right)^2\right)^x = \left(\frac{3}{2}\right)^3$$

$$\left(\frac{2}{3}\right)^{2x} = \left(\frac{3}{2}\right)^3$$

POSTUP NÍŽE V POZNÁMCE

$$\left(\frac{3}{2}\right)^{-2x} = \left(\frac{3}{2}\right)^3$$

$$-2x = 3 \quad /:(-2)$$

$$x = -\frac{3}{2}$$

Poznámka:

$$\left(\frac{3}{2}\right)^{-2x} = \frac{1}{\left(\frac{3}{2}\right)^{2x}} = \frac{1}{1} \cdot \left(\frac{2}{3}\right)^{2x} = \left(\frac{2}{3}\right)^{2x}$$

$$\left(\frac{2}{3}\right)^{2x} = \frac{1}{\left(\frac{2}{3}\right)^{-2x}} = \frac{1}{\frac{1}{\left(\frac{2}{3}\right)^{2x}}} = \frac{1}{\frac{1}{1} \cdot \left(\frac{3}{2}\right)^{2x}} = \frac{1}{\left(\frac{3}{2}\right)^{2x}} = \left(\frac{3}{2}\right)^{-2x}$$

EXPONENCIÁLNÍ NEROVNICE

viz příklad "jako" exponenciální rovnice:

$$2^{x^2-6x-\frac{5}{2}} \geq 16\sqrt{2}$$

$x_1 = 7$

$x_2 = -1$

$(x-7)\ (x-(-1))$

$(x-7)\ (x+1)$

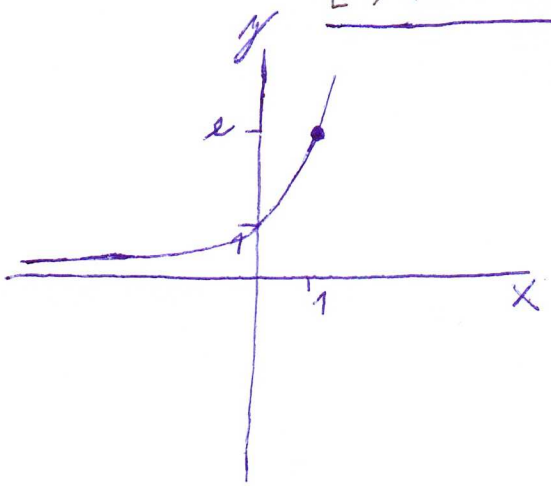
$x_0 = 7$

$x_0 = -1$

VÝSLEDEK

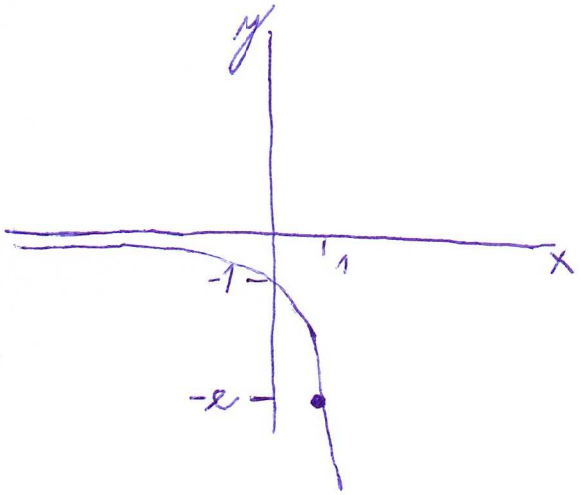
Tyto nulové body se dávají na číselnou osu.

EXPONENCIÁLNÍ FCE

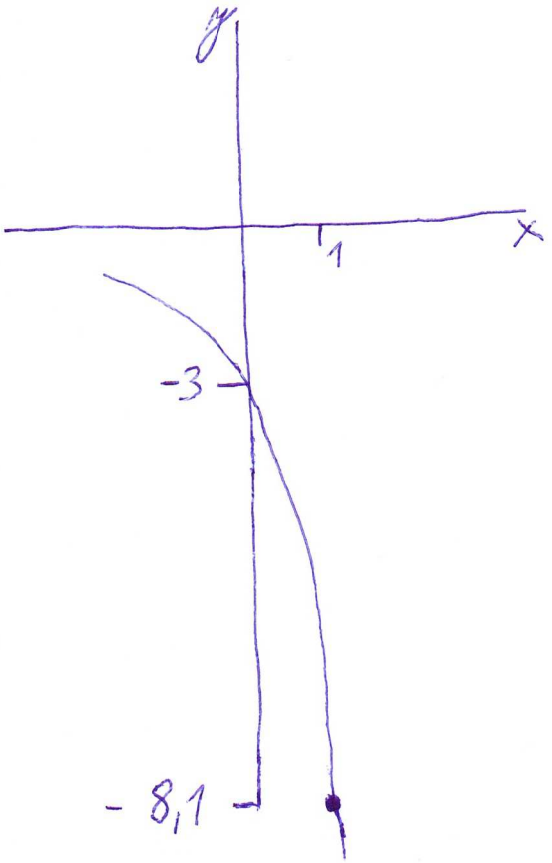


$e \doteq 2,7$

$y = a \cdot e^x$



$y = -a \cdot e^x$



$y = -3e^x$

$P_y = ?\quad x = 0$

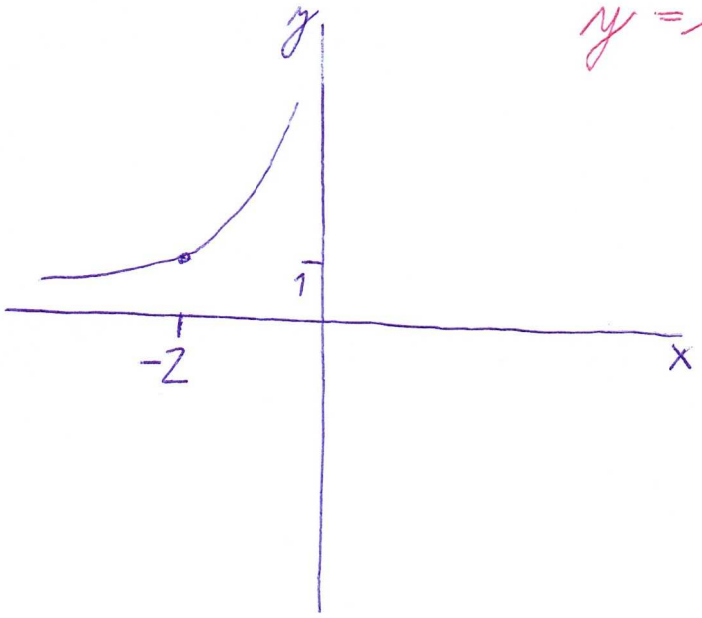
$y = -3e^0$

$y = -3 \cdot 1$

$y = -3$

$P_y\ [0, -3]$

$3 \cdot (-2,7) = -8,1$



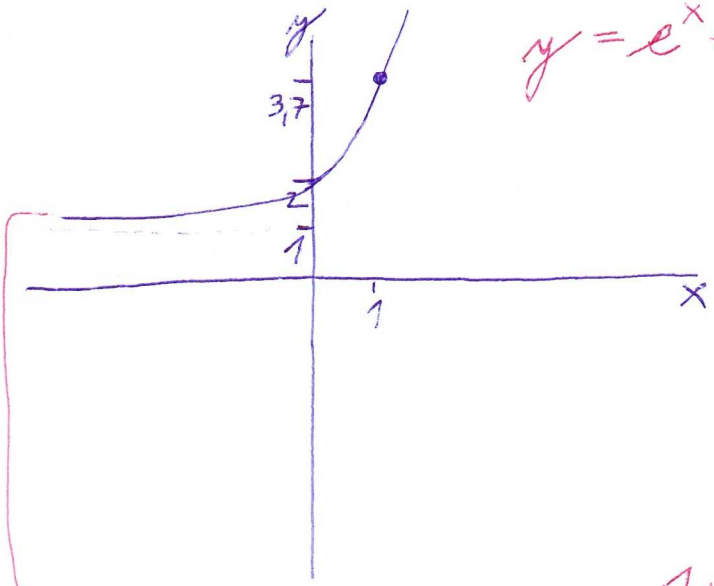
$y = e^{x+2}$

$y = e^{x+c}$

POSUN

$c > 0$ C KLADNÉ ←

$c < 0$ C ZÁPORNÉ →



$y = e^{x} + 1$

$y = e^{x} + b$

POSUN

$b > 0$ B KLADNÉ ↑

$b < 0$ B ZÁPORNÉ ↓

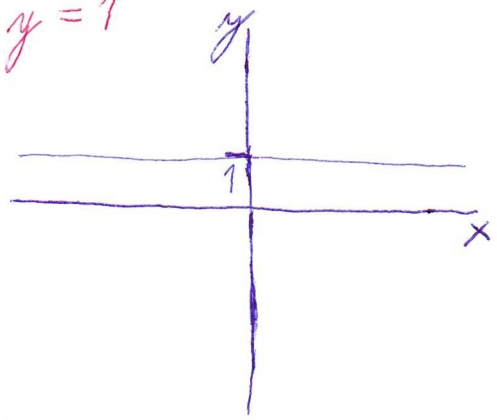
$1 + 1 = 2$

$2,7 + 1 = 3,7$

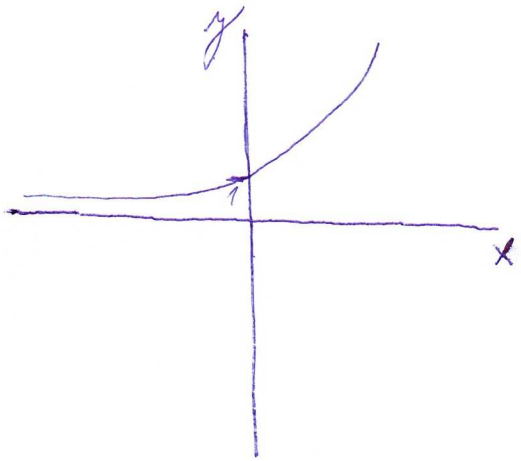
e^x se nikdy nedostane v oboru hodnot pod jedničku

$$y = d^x$$

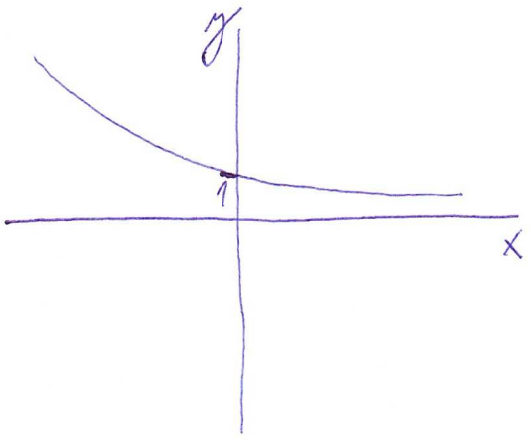
$y = 1^x$



$d = 1$



$d > 1$ d - libovolné číslo

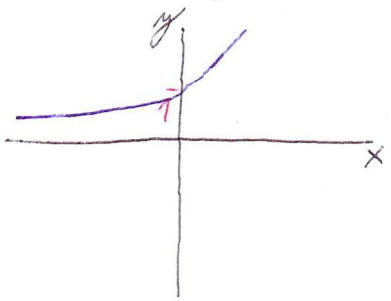


$0 < d < 1$ OTOČENÍ

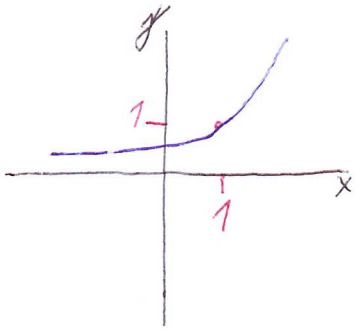
PŘÍKLAD: $f: y = -(2^{x-1}) - 3$

Je co je v závorce je v exponentu $\Rightarrow$ EXPONENCIÁLNÍ FUNKCE

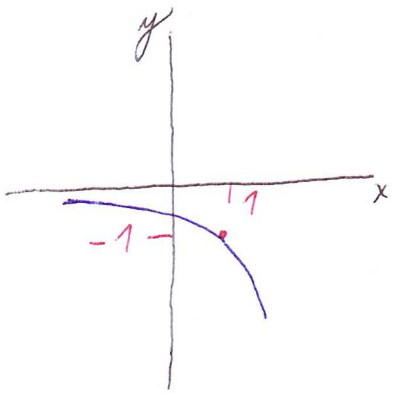
1) $y = 2^x$



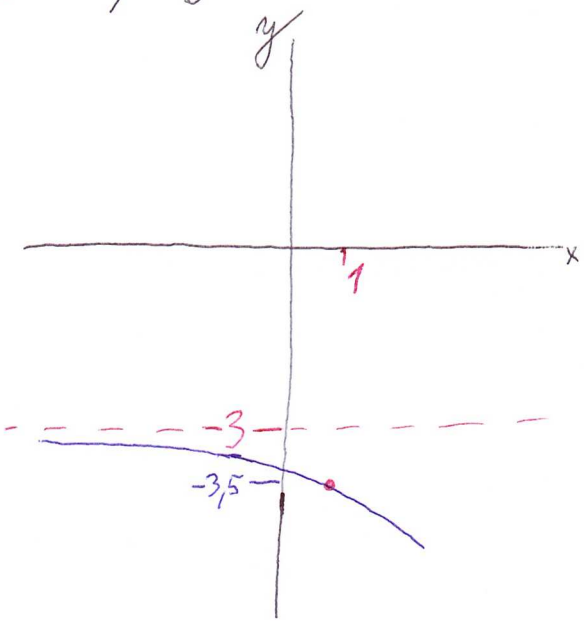
2) $y = 2^{x-1}$



3) $y = -(2^{x-1})$



4) $y = -(2^{x-1}) - 3$



$P_y = ?$ $x = 0$

$y = -(2^{0-1}) - 3$

$y = -(2^{-1}) - 3$

$y = -\left(\frac{1}{2^1}\right) - 3$

$y = -\frac{1}{2} - \frac{3}{1}$

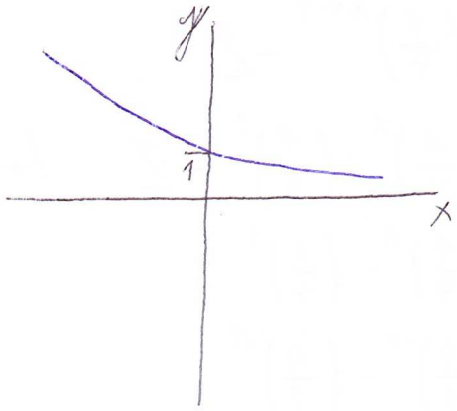
$y = \frac{-1-6}{2}$

$y = -\frac{7}{2}$

$P_y = [0, -\frac{7}{2}]$

PŘÍKLAD $f: y = 2 \cdot \left(\frac{1}{4}\right)^{x+1} - 2$

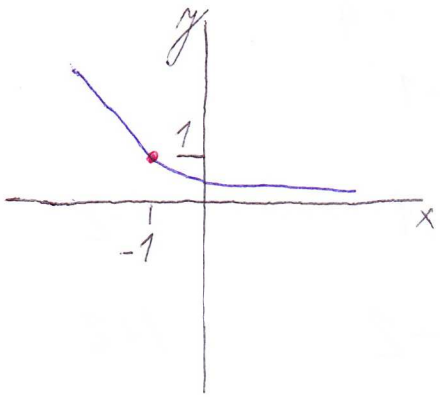
1) $y = \left(\frac{1}{4}\right)^x$



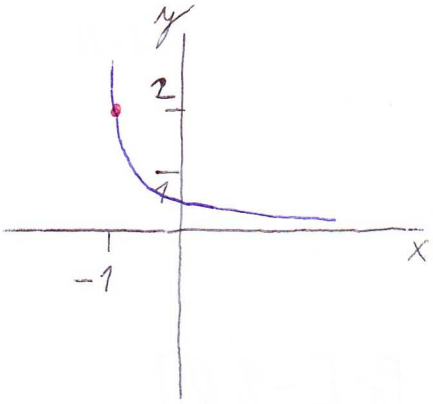
$y = d^x$

$0 < d < 1$

2) $y = \left(\frac{1}{4}\right)^{x+1}$

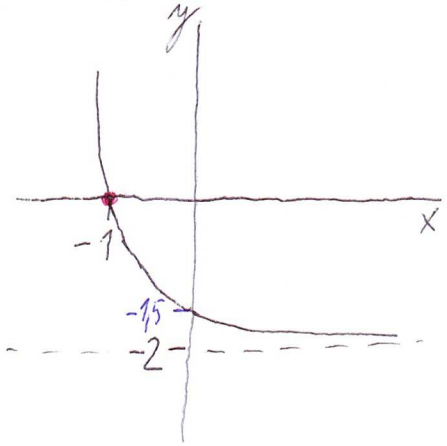


3) $y = 2 \cdot \left(\frac{1}{4}\right)^{x+1}$



STRMĚJŠÍ

4) $y = 2 \cdot \left(\frac{1}{4}\right)^{x+1} - 2$



$P_y = 2 \quad x = 0$

$y = 2 \cdot \left(\frac{1}{4}\right)^{0+1} - 2$

$y = \frac{2}{4} - \frac{2}{1}$

$y = \frac{2-8}{4} = \frac{-6}{4} = -\frac{3}{2}$

$P_y \left[0, -\frac{3}{2}\right]$

$P_x = ?$ $y = 0$

$$0 = 2 \cdot \left(\frac{1}{4}\right)^{x+1} - 2$$

$$0 = 2 \cdot \left(\frac{1}{4}\right)^{x} \cdot \left(\frac{1}{4}\right)^{1} - 2 \quad /+2$$

$$2 = 2 \cdot \left(\frac{1}{4}\right)^{x} \cdot \left(\frac{1}{4}\right)^{1}$$

$$2 = 2 \cdot \left(\frac{4}{1}\right)^{-x} \cdot \left(\frac{4}{1}\right)^{-1}$$

$$4^{\frac{1}{2}} = 4^{\frac{1}{2}} \cdot 4^{-x} \cdot 4^{-1}$$

$$4^{\frac{1}{2}} = 4^{\frac{1}{2} - x - 1}$$

$$\frac{1}{2} = \frac{1}{2} - x - 1 \quad /\cdot 2$$

$$1 = 1 - 2x - 2 \quad /+2$$

$$3 = 1 - 2x \quad /-1$$

$$2 = -2x \quad /\cdot(-1)$$

$$-2 = 2x$$

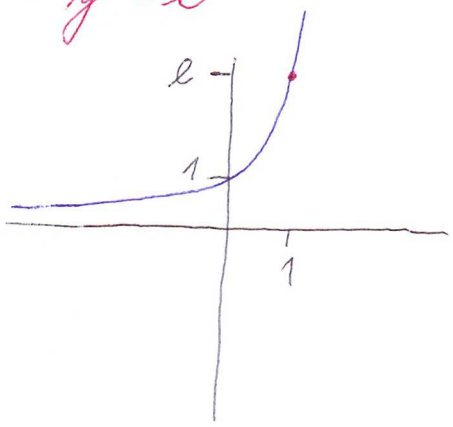
$$2x = -2 \quad /:2$$

$$x = -1$$

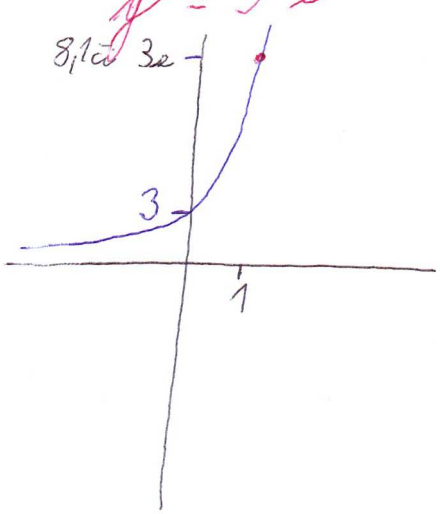
$$P_x\,[-1, 0]$$

SESTROJTE GRAFY EXPONENCIÁLNÍCH FUNKCÍ

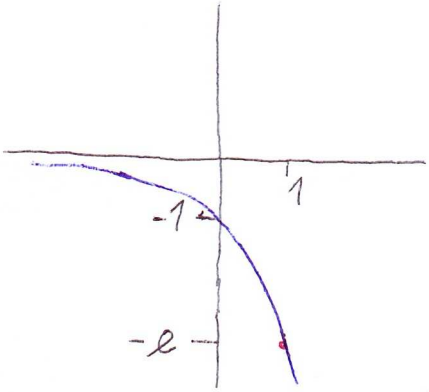
$y = e^x$



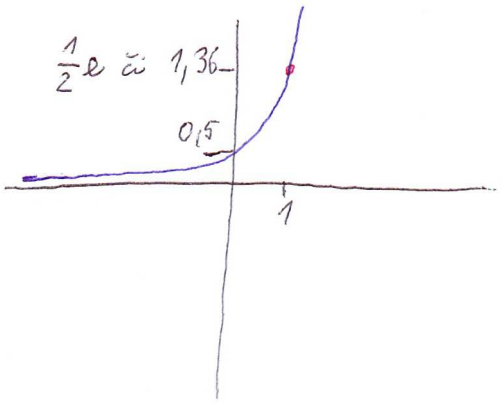
$y = 3 \cdot e^x$



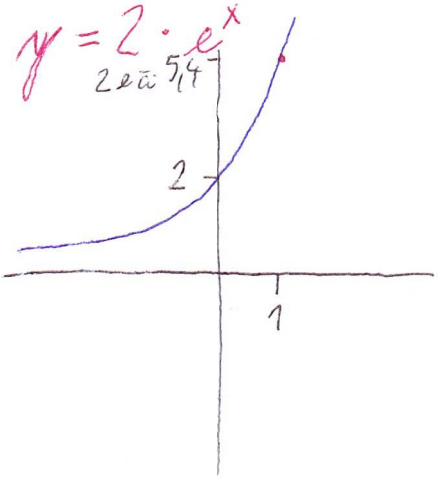
$y = -(e^x)$



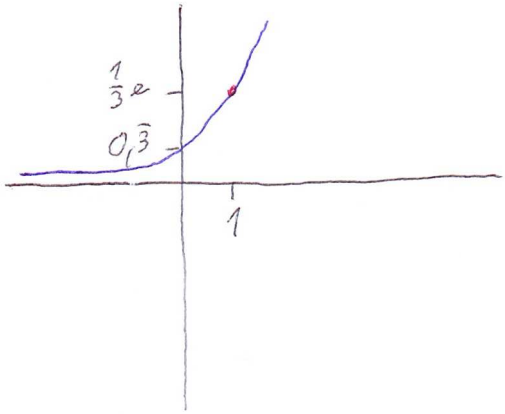
$y = \frac{1}{2} \cdot e^x$



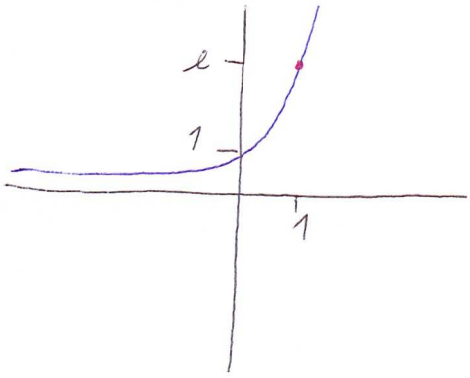
$y = 2 \cdot e^x$



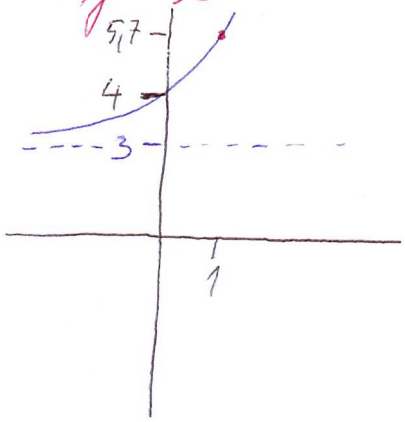
$y = \frac{1}{3} e^x$



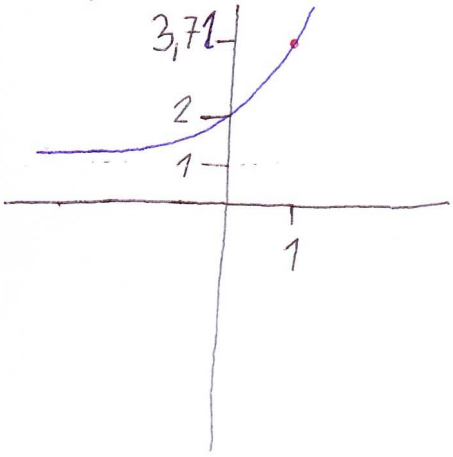
$y = e^x$



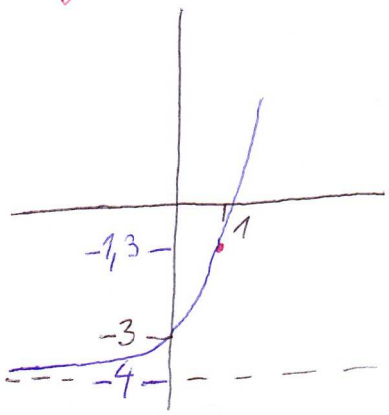
$y = e^x + 3$



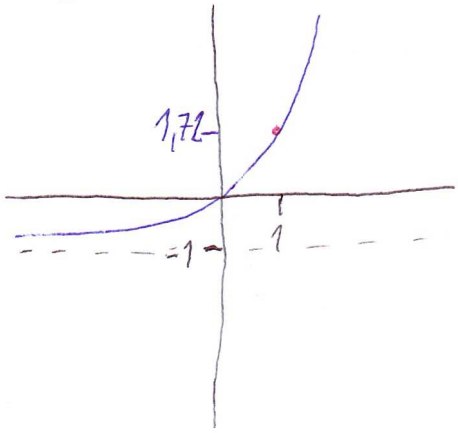
$y = e^x + 1$



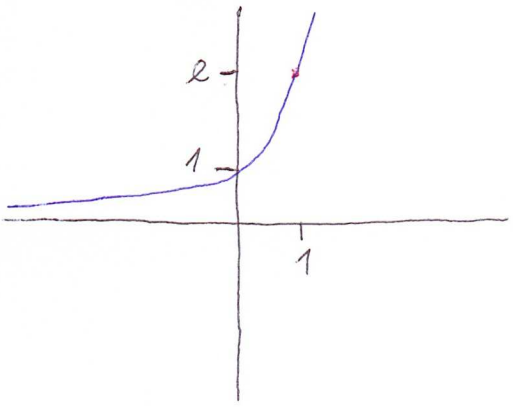
$y = e^x - 4$



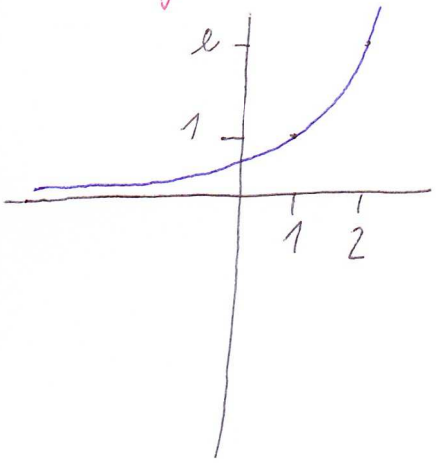
$y = e^x - 1$



$y = e^x$



$y = e^{x-1}$



$y = e^{x+2}$